Overview

Course Overview

Sample Proofs
Our Website: Planets!

- Class goals.
- Course outline.
- Meet the UTAs!
- Collaboration policy.
- Assignments, dates and deadlines.
- Attendance policy.
- Recitations.
- TA hours.
Other sites

- Campuswire: Best way to get quick answers. Key announcements there, too.
- Gradescope: Handins, homework grading.
- Overleaf (optional): LaTeX without installation.
Ethics in Discrete Math

- Two STAs this semester. Why?

- Math often seen as a “neutral” or “pure”. It’s more complicated than that.

- Math becomes relevant when it is applied to the real world. Doing so always requires simplifications.

- Issues arise via: (1) flawed assumptions when bridging between theory and reality, (2) and ethical flaws in understanding the “end-goal” application, and more.

- Keep uses in mind. The largest employer of mathematicians in the US is the NSA, which has clear ethical implications.

- We’ll be asking you to consider potential ethical implications of the topics we cover and the importance of considering issues in advance.
Odd times odd

- Poll. How approach a problem like this one?
- Check a few cases to see if you believe it.
  $3 \times 5 = 15$, $7 \times 3 = 21$. One times anything is the same, so, if it was odd, it stays odd. So far so good.
- Go to definitions. What does odd actually mean, mathematically? A number is odd if it can be written $2k + 1$ for an integer $k$.
- Use definitions to express the problem.
  We have two odd numbers: $2k_1 + 1$, $2k_2 + 1$. What can we say about their product?

\[
(2k_1 + 1)(2k_2 + 1) = 4k_1 k_2 + 2k_1 + 2k_2 + 1
\]
\[
= 2(2k_1 k_2 + k_1 + k_2) + 1
\]
\[
= 2k_3 + 1,
\]

Since $k_3 = 2k_1 k_2 + k_1 + k_2$ is an integer, the product is odd.
Bad “proof”

Each step must be done carefully to avoid going off the rails.

Pick any $y$ and let $x = 2y$

Multiply by $-x$

Add $2x^2$

Subtract $2xy$

Factor

Cancel common terms

Conclusion: Math is over. If we can conclude $1 = 2$, we can conclude *anything*.
Proof by contradiction

- If $n^2$ is even, then $n$ is even.

We are given that $n^2$ is even. Let’s assume that $n$ is odd. Earlier in the lecture, we showed that the product of two odds is odd. That implies that $n^2$ must be odd. But, $n^2$ can’t be both odd and even, so that’s a contradiction. That means the negation of our most recent assumption must be true. In other words, $n$ must have actually been even.

The basic idea is that if any time $a$ happens then $b$ must happen and we know $b$ didn’t happen, well, then $a$ couldn’t have happened either. After all, if $a$ had happened, it would have made $b$ happen. But, $b$ didn’t happen, so $a$ couldn’t have happened.
Deeper proof by contradiction

- √2 is irrational.

Let’s assume it is rational. That means √2 = p/q where p and q are integers. Furthermore, we can assume p/q is in lowest terms so they have no factors in common. Squaring both sides, we get 2 = p²/q² or 2q² = p². Since q² is an integer, and p² is an integer times 2, p² is even. By a similar argument to the one about odd squares, that means p must be even. If p is even, p² must be divisible by 4. Since 2q² is divisible by 4, q² must be divisible by 2. That means both p and q are even. But, then p/q is not in lowest terms, which we had already concluded. Because we’ve reached a contradiction, we can conclude that our most recent assumption must be false. That is, √2 must not be rational.